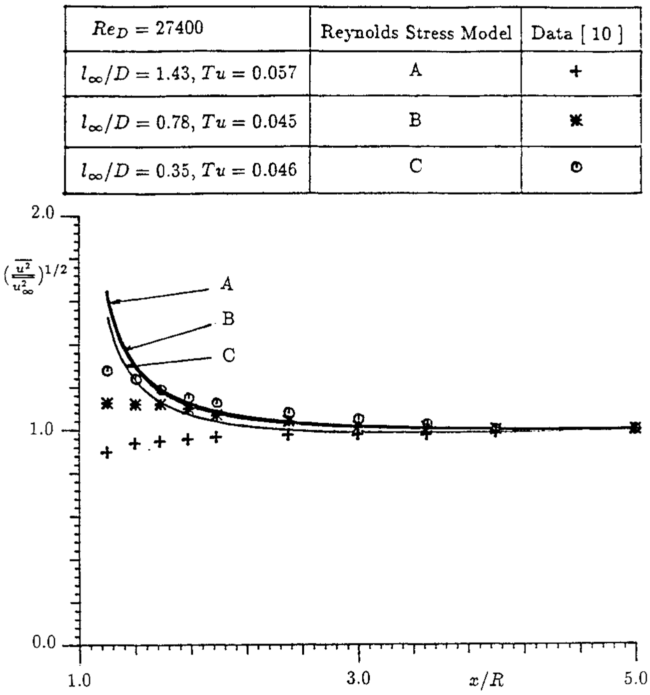

Fig. 3 Comparison of turbulent kinetic energy on the stagnation streamline for large freestream turbulence length scale.

assumption that the flow is locally homogeneous, which is not the case when the length scale is on the order of or larger than the body dimension.

In order to define the entire stagnation streamline process, an inner (boundary-layer) set of equations for the mean velocity and Reynolds stresses needs to be solved and matched to the outer flow solution in a manner similar to that done by Strahle et al.[4] for the k-ϵ model. Alternatively, the entire streamline flow from the surface to the far freestream could be directly solved with equations that contain diffusional-transport terms, which are important near the surface. This method was used by Hijikata et al.[5] with their three-equation model. Profiles for the mean velocity, kinetic energy, etc., obtained from the inner part of the stagnation streamline solution could then be used as initial conditions for a boundary-layer calculation on the surface away from the stagnation point. Taulbee et al.[11] used the near-surface profiles from the stagnation streamline solution to initiate a k-ϵ model boundary-layer calculation to obtain heat-transfer distributions on gas turbine blades.

In summary, it is stated that the two-equation turbulence model, which uses a gradient hypothesis for the Reynolds stresses, is not applicable for determining turbulence in stagnating flow. The Reynolds stress model of Lumley[9] gives good results when the length scale is small compared to the body dimension. Single-point closure models are not applicable when the length scale is as large or larger than the body dimension.

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Simple Expressions for Higher Vibration Modes of Uniform Euler Beams

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THE vibration modes and frequencies of uniform Euler beams for various boundary end conditions are traditionally expressed in terms of $\sin\beta_n x$, $\cos\beta_n x$, $\sinh\beta_n x$, and $\cosh\beta_n x$ functions (see, e.g., Refs. 1-3). For modes above the second, however, the numerical evaluation of these modes requires an increasingly large number of significant figures to be kept in order to distinguish small differences between $\sinh\beta_n x$ and $\cosh\beta_n x$. The present note, based on ideas presented in Ref. 3, gives simple expressions for arbitrarily high-order modes and fre- quencies of a uniform Euler beam. These expressions not only allow simple numerical calculation but also help to identify the physical nature of the modes.

Consider, for example, a free-free uniform Euler beam. The modes $\phi_n(x)$ and frequencies ω_n are traditionally expressed as

$$\phi_n(x) = \cosh\beta_n x + \cos\beta_n x - \alpha_n(\sinh\beta_n x + \sin\beta_n x) \qquad (1)$$

$$\omega_n = \beta_n^2 \sqrt{EI/m\ell^4} \qquad (2)$$

where β_n is the solution of the transcendental equation shown below, and α_n is related to β_n as indicated in Eq. (4).

$$\cos\beta_n = (1/\cosh\beta_n) \qquad (3)$$

$$\alpha_n = \frac{\cosh\beta_n - \cos\beta_n}{\sinh\beta_n - \sin\beta_n} \qquad (4)$$

Received Jan. 26, 1988.

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Table 1 Euler beam elastic mode shape parameters

Boundary condition[a]	β_n	θ	A	B
SS-SS	$n\pi$	0	0	0
CL-FR	$(n - 1/2)\pi$	$-\pi/4$	1	$(-1)^{n+1}$
CL-CL	$(n + 1/2)\pi$	$-\pi/4$	1	$(-1)^{n+1}$
FR-FR	$(n + 1/2)\pi$	$+3\pi/4$	1	$(-1)^{n+1}$
SS-CL	$(n + 1/4)\pi$	0	0	$(-1)^{n+1}$
SS-FR	$(n + 1/4)\pi$	0	0	$(-1)^{n}$

[a]SS = simply supported, CL = clamped, and FR = free.

In the above, EI is the bending stiffness, m is the mass per unit length, ℓ is the beam length, and x is the beam coordinate, nondimensionalized with respect to length ℓ, and varying between 0 and 1. The mode shape $\phi_n(x)$ satisfies the condition

$$\int_0^1 \phi_n^2(x)\,dx = 1 \tag{5}$$

Following a suggestion in Ref. 3, Eq. (1) can be rearranged, by adding and subtracting $\sinh\beta_n x$, into the form

$$\phi_n(x) = \cos\beta_n x - \alpha_n \sin\beta_n x + e^{-\beta_n x} + (1 - \alpha_n)\sinh\beta_n x \tag{6}$$

Introducing Eq. (4), the $(1 - \alpha_n)$ term in Eq. (6) becomes

$$1 - \alpha_n = \frac{\cos\beta_n - \sin\beta_n - e^{-\beta_n}}{\sinh\beta_n - \sin\beta_n} \tag{7}$$

The solution of Eq. (3) for β_n is found by plotting the right- and left-hand sides and is given, for $n \geq 2$, to less than 0.01% accuracy as

$$\beta_n \approx (n + 1/2)\pi \tag{8}$$

For $n = 2$, one has $\beta_2 = 7.8540$, $\sinh\beta_2 = 1288.0$, $\exp(-\beta_2) = 0.000388$. Under these conditions and using Eq. (8), the relation Eq. (7) can be approximated as

$$(1 - \alpha_n) \approx \frac{-(-1)^n}{\sinh\beta_n} \approx (-1)^{n+1}\, 2e^{-\beta_n} \tag{9}$$

The last two terms of Eq. (6) then can be combined to give

$$e^{-\beta x}[1 - (-1)^{n+1} e^{-\beta_n}] + (-1)^{n+1} e^{-\beta_n(1-x)} \tag{10}$$

Since $\exp(-\beta_n) << 1$ and from Eq. (4) one has $\alpha_n \approx 1$, the mode shape given by Eq. (6) can be rewritten in the simple form as

$$\phi_n = \cos\beta_n x - \sin\beta_n x + e^{-\beta_n x} + (-1)^{n+1} e^{-\beta_n(1-x)} \tag{11}$$

Equation (11) represents a shifted sinusoidal wave with small exponential end corrections. It is easily evaluated numerically, nodes and peaks are readily identified, and it is valid for any mode $n \geq 2$. Even for $n = 1$, it gives only a 0.7% error in amplitude at the midpoint.

Similar expressions can be developed for other boundary conditions. The higher vibration modes of uniform Euler beams for $n \geq 2$ can then be simply expressed as

$$\phi_n(x) = \sqrt{2}\sin(\beta_n x + \theta) + Ae^{-\beta_n x} + Be^{-\beta_n(1-x)} \tag{12}$$

where the constants β_n, θ, A, and B are given in Table 1 for some common boundary conditions. The corresponding frequencies are given by Eq. (2). All modes are normalized such that Eq. (5) holds. These modes also apply for $n = 1$ with less than a 1% error, except for the clamped-free case. Not only are these modes easily evaluated numerically, but they are immediately identified as simple shifted sinusoidal waves with small, confined, exponential edge-zone corrections to match the required boundary conditions at each end. In terms of the mode wavelength $\lambda = 2\pi/\beta_n$, the edge-zone correction dies out to 5% of its maximum end value (i.e., $\beta_n x = 3$) in roughly half a wavelength. The forms of Eq. (12) can be very useful in performing large multimode Rayleigh-Ritz type analyses for beams, plates, and shells with different boundary conditions. They also show clearly that standing waves of a beam have the same sinusoidal structure as traveling waves except for small edge-zone regions near the ends where reflections and absorptions occur. The analysis developed here may also prove useful in dealing with Timoshenko beams and related shear-type structures. An extensive survey of related work and applications of edge effects has been summarized by Elishakoff.[4] Also, the form of Eq. (11) has been presented earlier by Dowell[5] for the clamped-free and free-free beam cases.

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Active Vibration Isolation by Polymeric Piezoelectret with Variable Feedback Gains

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ACTIVE vibration involves the cancellation of unwanted oscillation with an equal and opposite excitation that is artificially generated by an arrangement of active vibration control devices. Numerous active control systems using electromagnetic,[1] pneumatic,[2] hydraulic,[3] and viscoelastic[4] force generators have been investigated. In recent years, piezoelectrets were also used as active actuators for active vibration control of distributed parameter systems[5-8] because of their potential applications in large flexible and precision space structures.

This paper presents a theoretical and experimental study of an active piezoelectric vibration isolation technique using polymeric piezoelectric polyvinylidence fluoride (PVDF) with variable feedback gains. Injecting feedback voltage into a PVDF isolator results in a thickness change of the isolator due to its converse piezoelectric effect. If this thickness change is adjusted 180 deg out of phase with the base excitation, the PVDF isolator (with appropriate feedback gain) can effectively cancel any disturbance from the base. Theory on active piezoelectric vibration isolation with general mechanical and

Received Nov. 30, 1987; revision received April 21, 1988.

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